Partitioning networks into colorful components

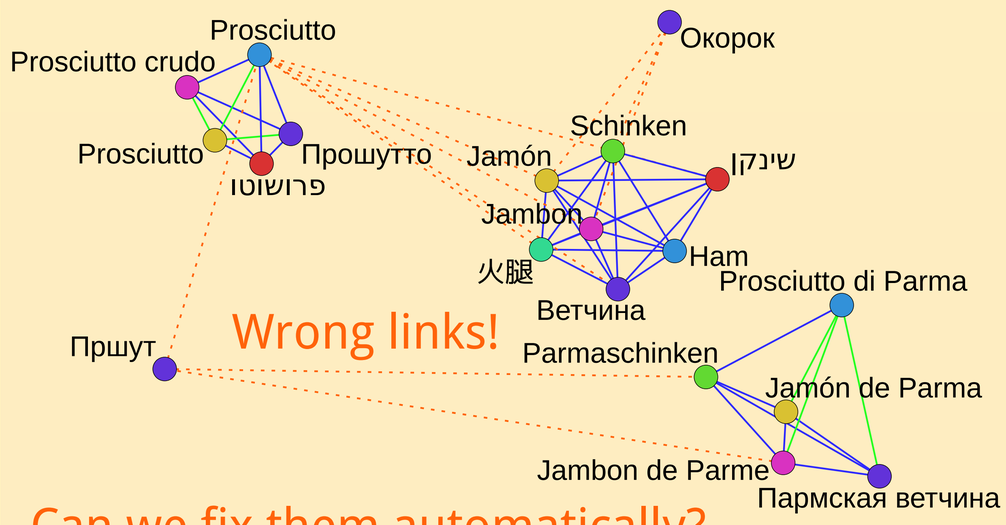
or: how to fix Wikipedia interlanguage links

Falk Hüffner
falk.hueffner@tu-berlin.de
joint work with: Sharon Bruckner Christian Komusiewicz Rolf Niedermeier

Problem

Wikipedia interlanguage links

Wikipedia interlanguage network



Can we fix them automatically?

Observation

If there is a link path from a word in some language to a different word in the same language, then at least one of the links on this path is wrong: e.g.

Ham – Jambon – Schinken – Prosciutto

Optimization problem

Colorful Components
Instance: A network where each node has a color.
Task: Delete a minimum number of links such that all connected components are colorful, that is, contain each color only once.

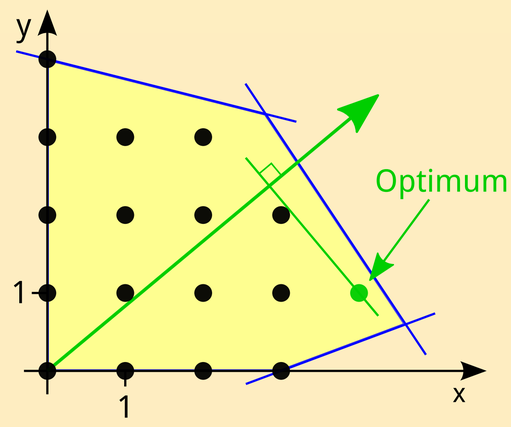
Obstacle

Colorful Components is NP-hard! Thus, there is probably no efficient algorithm that always gives an optimal solution.

Approaches

Integer Linear Programming (ILP)

Idea: Express the problem using linear constraints and a linear objective and use an ILP solver.

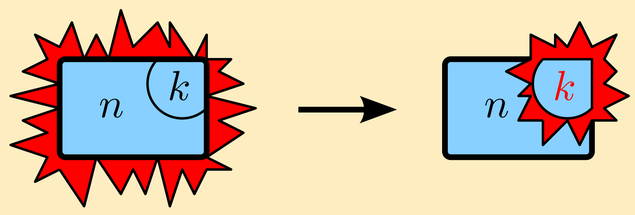


Parameterized complexity

Idea: Analyze the running time not only with respect to the problem size n, but also with respect to some parameter k, e. g. the number of colors c or the number of link deletions d.

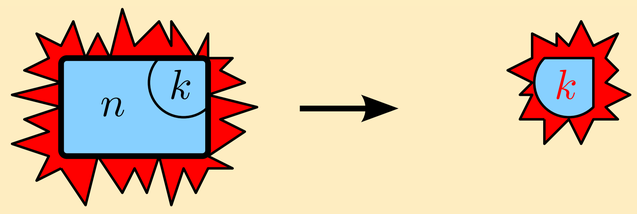
Fixed-Parameter Algorithms (FPT)

Idea: Try to confine the combinatorial explosion to some parameter k.



Data reduction/Kernelization

Idea: Remove redundant parts of the input, such that the size of the remaining instance depends only on some parameter k.



Methods

Colorful Components ILP formulation

Use binary variables e_{uv} for each node pair (u, v), where $e_{uv} = 1 \Leftrightarrow u$ and v are in the same cluster.

maximize $\sum_{\{u,v\} \in E} e_{uv}$
subject to $e_{uv}=0$ when color(u)=color(v)
$e_{uv}+e_{uw}-e_{vw} \leq 1$ for nodes u, v, w

Improvements

Use cutting planes and row generation.

FPT algorithm for Colorful Components

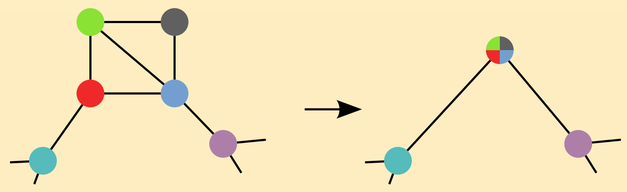
Idea: Find a path between equally-colored nodes (bad path) and recursively try deleting each link.

Improvement

Idea: If there is a node with at least three neighbors, we can find a bad path with at most $c-1$ links; otherwise, the instance is easy.

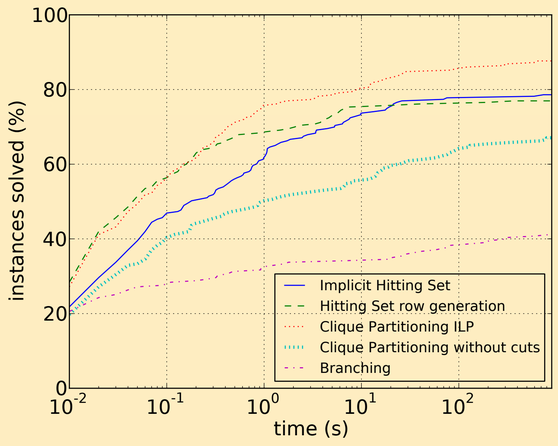
Data reduction for Colorful Components

Idea: Join nodes that cannot be split.



Results

Running time for random instances



Theorem

Colorful Components can be solved in $O((c-1)^d \cdot n^2)$ time.

Implication

We can find optimal solutions with useful running time guarantees: if the number of colors and the number of links deleted are small, we can solve the problem quickly, even if the network is very large.

Theorem

Colorful Components has a kernel with at most $c \cdot d$ nodes.

Implication

We have an efficient preprocessing with useful quality guarantees, which can be combined with any other approach, be it exact, approximative or heuristic.

Final result for the Wikipedia network

Using a combination of data reduction and the ILP formulation, we can optimally solve in 80 minutes Colorful Components for the Wikipedia interlanguage link network of the 30 most popular languages with 11,977,500 nodes and 46,695,719 links. The largest connected component has 1,828 nodes and 14,403 links. We find 618,660 links to be deleted and 434,849 to be added.

Outlook: Other applications

- Matching of products from online store
- Matching profiles from different social networks
- Multiple sequence alignment

Outlook: Model extensions

- Demand better connected clusters
- Allow some duplicates per cluster
- Allow link weights

Outlook: Algorithmic improvements

- Column generation
- More data reduction rules
- Heuristics

Reference

Sharon Bruckner, Falk Hüffner, Christian Komusiewicz, Rolf Niedermeier, Sven Thiel, and Johannes Uhlmann: *Partitioning into colorful components by minimum edge deletions*. In Proceedings of the 23rd Annual Symposium on Combinatorial Pattern Matching (CPM '12), Helsinki, Finland. July 2012. Volume 7354 in Lecture Notes in Computer Science, pages 56–69, Springer, 2012.