Data Reduction, Exact, and Heuristic Algorithms for Clique Cover

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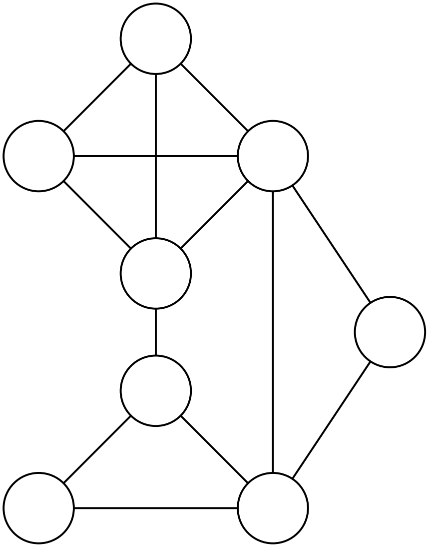
Clique Cover

Definition

(Edge) Clique Cover

Input: An undirected graph $G = (V, E)$.

Task: Find a minimum number k of cliques such that each edge is contained in at least one clique.



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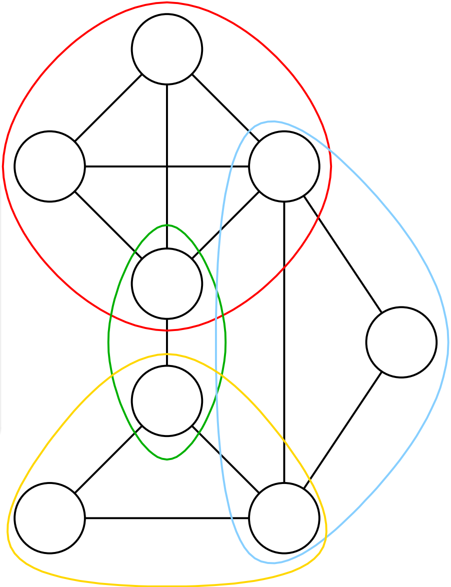

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Also known as

- KEYWORD CONFLICT [KELLERMAN, IBM 1973]
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Properties

- NP-complete [GAREY&JOHNSON 1979]
- NP-hard to approximate to constant factor [AUSIELLO ET AL. 1999]

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ANNOTATED CLIQUE COVER

- Edges can be marked as covered
- Only uncovered edges have to be covered by cliques

Simple Data Reduction Rules for Clique Cover

Rule 1

Remove isolated vertices and vertices that are only adjacent to covered edges.

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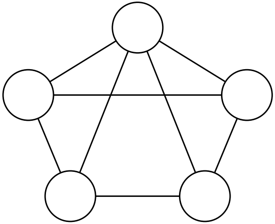
Rule 2

If an edge $\{u, v\}$ is contained only in exactly one maximal clique C, then add C to the solution, mark its edges as covered, and decrease k by one.



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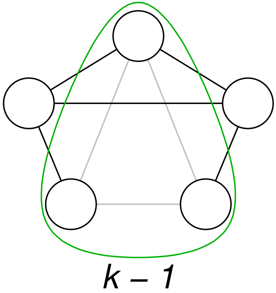


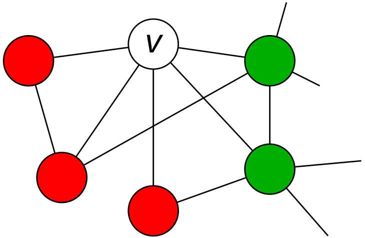
Prisoner/Exits Reduction Rules for Clique Cover

Partition the neighborhood of a vertex v into:

- prisoners p with $N(p) \subseteq N(v)$ and
- exits x with $N(x) \setminus N(v) \neq \emptyset$.

Rule 4

If all exits have at least one prisoner as neighbor, then delete v.



Fixed-Parameter Tractability of Clique Cover

Consider a Clique Cover instance with n vertices and k cliques allowed.

Theorem

After applying all reduction rules exhaustively, a Clique Cover instance has at most 2^k vertices, that is, Clique Cover has a problem kernel of size 2^k.

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Corollary

CLIQUE COVER *is fixed-parameter tractable with respect to the parameter* k*, that is, it can be solved in time* $f(k) \cdot n^{O(1)}$ *for some function* f *depending only on* k.

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Search-tree algorithm for CLIQUE COVER:

- Choose some uncovered edge e
- For each maximal clique C that contains e, mark all edges in C as covered, decrease k by one, and call the algorithm recursively

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- Horrible worst-case complexity...

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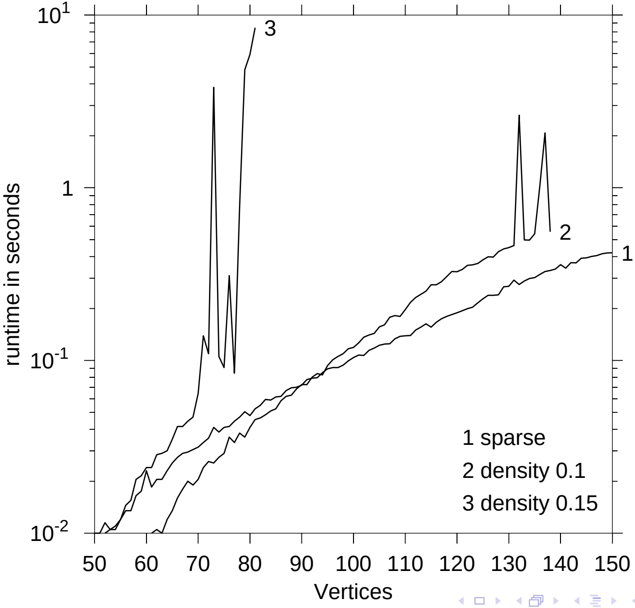
Results:

- Horrible worst-case complexity...

...but:

- Works nicely in practice when combined with data reduction rules.
- Can solve all instances in a benchmark from applied statistics within a second (up to 124 vertices and 4847 edges).
- Can solve sparse instances with hundreds of vertices and tens of thousands of edges within minutes.

Exact Algorithm for Clique Cover



Summary

- Data reduction rules can be successfully applied to CLIQUE COVER.
- An exact algorithm based on the data reduction rules and a search tree can solve many practically relevant instances.
- Further results in the paper: runtime improvement for a heuristic.

Open question

In the statistics application, it is also desirable to minimize the sum of clique sizes.

Question

Is there a solution that minimizes the sum of clique sizes, but not the number of cliques?