# Implementation Aspects of Data Reduction

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### Kernelization

Reduces a problem in polynomial time to a decision-equivalent, provably smaller one

### Data reduction rule

If applicable, reduces a problem in polynomial time to a smaller one, from whose solution an optimal solution to the original problem can be reconstructed.

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- But few sytematic studies on general NP-hard problems in the parameterized context

# Case Studies

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[Alber, Betzler & Niedermeier, Ann. Oper. Res. ’06]

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**Lesson**

Try all reduction rules, independent of proven effect.

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Solve Clique as Vertex Cover on the complement graph ($n \approx 1000$)

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Vertex Cover on planar graphs [Alber, Dorn & Niedermeier, Discrete Appl. Math.]

- 60-70 % reduction

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Cluster Editing [Böcker, Briesemeister & Klau, Algorithmica ’10]

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**Lesson**

Consider using parameter-dependent reduction rules.

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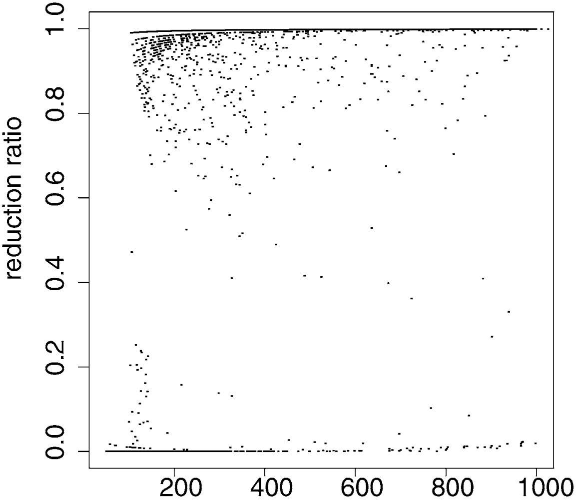

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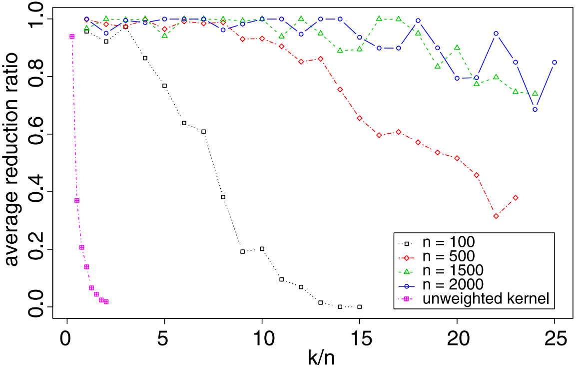

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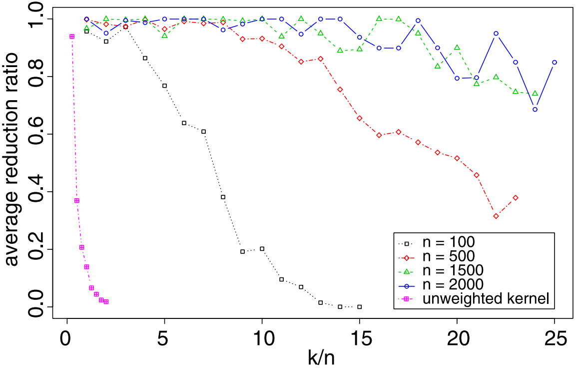


## Lesson

Consider solving a harder problem than the one you need to solve.

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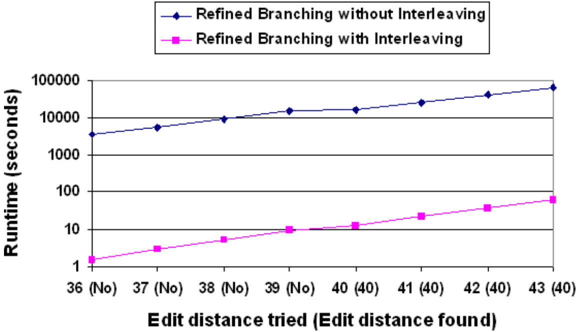


[Dehne et al., IWPEC ’06]

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## Possible solutions

1. Copy whole graph in each step
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3. Use an “undo” function for each branch or reduction that undos all changes.

# Persistent data structures

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In purely functional programming languages, all data structures are persistent.

E. g. persistent big-endian Patricia trees:

- $O(\log n + \deg v)$ neighborhood enumeration
- $O(\log n)$ edge test
- $O(\log n)$ edge insertion/deletion
- $O(\log n)$ vertex insertion
- $O(\log n \deg v)$ vertex deletion

# Persistent data structures

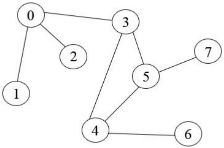
## Advantages

- No linear copy overhead
- Very easy to implement
- little error prone
- Quick and easy operations like intersection of neighbor sets

## Disadvantages

- Logarithmic overhead on all operations

# Implicit undo data structures




IM

$$\begin{array}{cc} & \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \begin{bmatrix} -1 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 2 & -1 & -1 & -1 & 0 & 0 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 2 & 1 & -1 & -1 & 0 \\ -1 & -1 & -1 & -1 & 2 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 2 & -1 & -1 \end{bmatrix} \end{array}$$

DEGREE

$$\begin{array}{cc} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 3 \\ 1 \\ 1 \\ 3 \\ 3 \\ 3 \\ 1 \\ 1 \end{bmatrix} \end{array}$$

AL

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LIST

$$\begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix} \end{array}$$

IDXLIST

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[Abu-Khzam, Langston, Mouawad & Nolan, FAW ’10]

# Implicit undo data structures

- $O(\deg v)$ neighborhood enumeration
- $O(1)$ edge test
- $O(1)$ edge insertion/deletion
- $O(1)$ vertex insertion
- $O(\deg v)$ vertex deletion

## Advantages

- Very little time overhead
- 5–10 times faster than simple adjacency list

## Disadvantages

- Large memory overhead
- Nontrivial graph modifications (e. g., edge contraction) become complicated

# Caching

**Example**

Keep a sorted map from vertex degree to the list of vertices of that degree.

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## Solution

Record for each edge $\{u, v\}$ the number of edges in the graph $G[N(u) \cap N(v)]$, using a priority queue.

[Gramm, Guo, Hüffner & Niedermeier, ACM J. Exp. Algorithmics ’08]

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When not using branching, even preprocessing that is not provably polynomial-time can help.

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## Result

Using non-optimality-preserving data reductions, a “kernel” guaranteeing approximation factor 1.5 can be found for Vertex Cover. [Asgeirsson & Stein, WEA ’07]

# Outlook

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- Graph data reduction language and framework
- Data reduction and enumeration