Implementing Fixed-Parameter Algorithms

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Real-world instances

Recommendation

Use real-world data.

- Easier to "sell"
- Analysis might lead to new insights and approaches
- More fun

Real-world instance sources

Databases

- Biological networks
- Social networks

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Web crawling

- DBLP coauthor graph
- Song similarity graph (last.fm)
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Cooperation

- Statistics visualization
- Power line network

Parameters

- solution size
- distance from tractable instances (e. g. treewidth)
- structural parameters (e. g. vertex cover size)

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Advertisement

Graphana (http://fpt.akt.tu-berlin.de/graphana/) is a tool that calculates parameters of graphs such as treewidth, connectivity, vertex cover size, cluster vertex deletion number, cluster editing number, h-index, degeneracy, feedback vertex set size, dominating set size, . . .

Randomly generated instances

Advantages of randomly generated instances

- Can have any number and size
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Types:

- Application oriented:
  - Phylogenetic trees
  - Web graphs
  - DNA sequences
- general

Example: Colorful Components

Colorful Components

Instance: An undirected graph $G = (V, E)$ and a coloring of the vertices $\chi : V \to \{1, \ldots, c\}$.
Task: Delete a minimum number k of edges such that all connected components are *colorful*, that is, they do not contain two vertices of the same color.
Parameter: k

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Random model:

- c: number of colors;
- n: number of vertices;
- p_v: probability that a component contains a vertex of a certain color;
- p_e: edge probability within component;
- p_x: edge probability between components.

A simple solver

Recommendation

Implement a solver that is as simple as possible.

Advantages

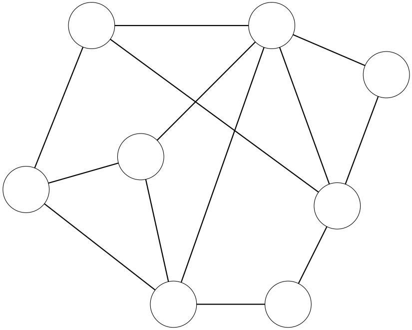
- first impression on what solutions look like
- base line for finding bugs

Typically simplest:

- Branching
- Integer Linear Programming (ILP)

Graph Bipartization

Graph Bipartization: Find a minimum size set of vertices in a graph whose removal results in the graph being bipartite.



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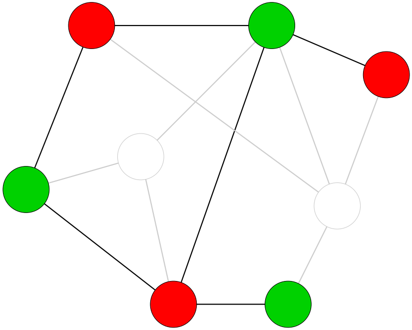

ILP for Graph Bipartization

$c_1, \ldots, c_n$: binary variables *(cover)*

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which can be expressed as an ILP constraint as

$$\text{s. t. } \forall \{v, w\} \in E : s_v + s_w + (c_v + c_w) \geqslant 1$$
$$\forall \{v, w\} \in E : s_v + s_w - (c_v + c_w) \leqslant 1$$

Implementation language

Recommendation

Use a high-level programming language.

Advantages

- more rapid development of typically exponential speedups, but only constant-factor slowdown
- persistent data structures allow simpler and less error-prone implementation of branching algorithms

Debugging

Recommendation

Verify that your solution is a solution.

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Use version control.

Data reduction

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Implement data reduction rules.

Advantages

- can be combined with approximation, heuristics, fixed-parameter, or other exact algorithms
- often very effective, even solve the whole instance
- normally, the more, the better

Heuristic speedups

Heuristic speedups in branching algorithms:

- Heuristic branching priorities
- Lower bounds

Benchmark set

Recommendation

Create a benchmark set using the randomized generator and parameter settings that match those measured in the real-world instances.

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- $c \in \{3, 5, 8\}$,
- $n \in \{60, 100, 170\}$,
- $p_v \in \{0.4, 0.6, 0.9\}$,
- $p_e \in \{0.4, 0.6, 0.9\}$,
- $p_x \in \{0.01, 0.02, 0.04\}$.

Comparison of algorithms

Time measurements depend on

- Machine
- Compiler options
- Program name
- Weather
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For exponential-time algorithms, averages of running time are useless.

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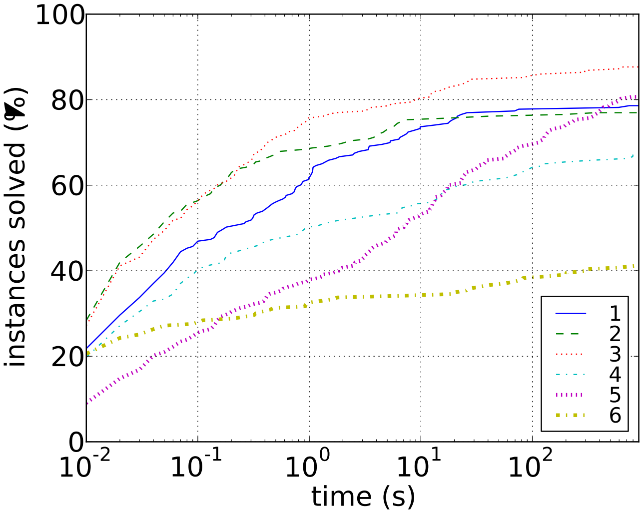
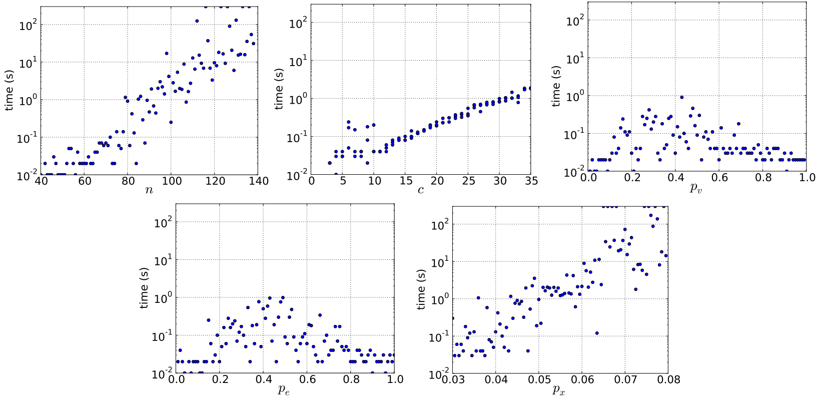
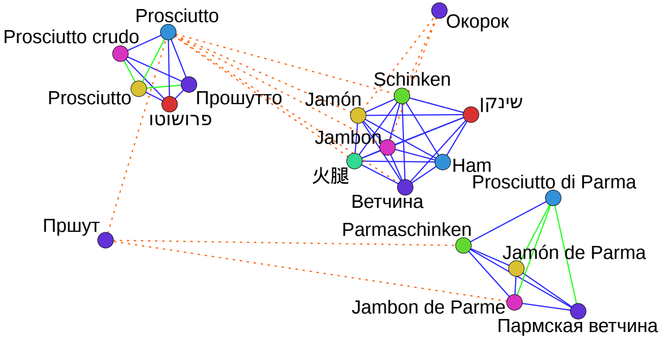


Evaluating one algorithm



Evaluate solution quality

Solutions are optimal, but might not match real-world truth due to model deficiencies.



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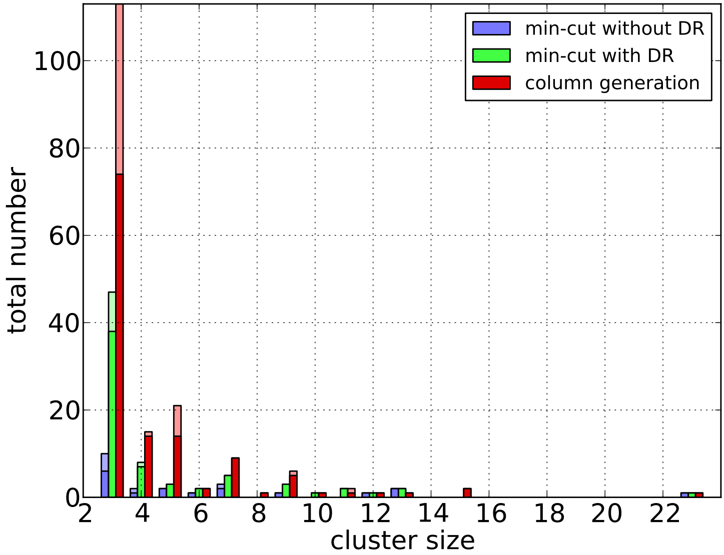

Finally

Recommendation

Make source and data available under a free license.